

scheme with full instructions for the collection of the material desired, and for the counting, measuring, &c., of fishes, fry and eggs. If these schemes are approved by the General Council, the president or secretary of the Central Bureau is authorised to recommend the leaders of the fishery experiments in all parts of the seas concerned to select the material desired from every catch, and to measure, mark and register it in the manner prescribed. Messrs. A. and B. will thus receive preparations or specimens of cod, halibut, &c., of such size and stage of development as they wish to study from every corner of the area investigated. Likewise, they will get analytical data of the salinity, temperature of the water or samples of plankton, stomach contents, gases contained in the water or in the bladder, &c.

The field of research of each specialist will thus be immensely enlarged. Another advantage is that material of purely scientific value can be distributed to public and private institutions, museums, &c., in the different countries.

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### Gibbs's Thermodynamical Model.

IN Maxwell's "Theory of Heat" (p. 207) is a drawing showing some of the principal lines on a thermodynamical model suggested by Prof. J. Willard Gibbs, of Yale University. I have been told that Prof. Maxwell had two of these models constructed, one of which remained at Cambridge, England, the other being sent to Prof. Gibbs at Yale. There is also a copy of this model at Clark University, Worcester, the only one which I have seen. While there may be others in existence, these are the only ones which I have known of, and I suspect that very few have ever been constructed.

This year, in connection with a course in thermodynamics, two of my pupils are attempting to construct one of these models, but are met by various serious difficulties, which may interest others. In the diagram to which I refer, the directions chosen for the different co-ordinates are not immediately evident. Even by the aid of the description in the text, I have not been able to locate them satisfactorily. In the attempt so to do, I have been guided by the following general considerations. Using Maxwell's notation, in which  $v$ =volume,  $p$ =pressure,  $t$ =absolute temperature,  $e$ =energy,  $\phi$ =entropy, the equation connecting these quantities is

$$td\phi = de + pdv,$$

which, transposed, gives

$$de = td\phi - pdv = \frac{\partial e}{\partial \phi} d\phi + \frac{\partial e}{\partial v} dv,$$

the differential equation of the thermodynamical surface of which the co-ordinates are the entropy, volume and energy, and the slope of which at each point in the principal directions gives the temperature and pressure, by the identities

$$t = \frac{\partial e}{\partial \phi}; \quad p = -\frac{\partial e}{\partial v}.$$

These are subject to the conditions that  $t$  is always positive, and  $p$  is usually positive, always so for the gaseous state, usually for the liquid and solid states.

If, then,  $e$  is taken vertically downward, and  $v$  and  $\phi$  horizontal, passing along a section of the surface by a plane of constant volume, in the direction of increasing entropy, the slope will always be downward, and generally convex, as the addition of heat, that is, energy to a substance at constant volume increases its entropy, and generally its temperature, never decreasing it. A section by a plane of constant entropy will have a slope in the direction of increasing volume, which is in general upward, corresponding to a positive pressure, and in all parts of the model referring to stable states of the substance this will be convex, since increase of volume is then accompanied by decrease of pressure.

I have attempted to determine the choice of co-ordinates by the properties of the critical state. In the two diagrams the broken line separates the parts representing stable or homogeneous states from parts representing unstable or non-homogeneous states. In the pressure-volume diagram, lines of constant pressure, volume, entropy and temperature are drawn. On the other are drawn lines of constant pressure and temperature, taken from Maxwell. In both diagrams these lines are tangent to the broken line. In Fig. 1 the line  $v$ =const. cuts sharply through the broken line.

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I have attempted to find the behaviour of a line of constant entropy in the following way.

For a substance following van der Waals's equation

$$\left(p + \frac{a}{v^2}\right)(v - b) = Rt,$$

the equation of an isentropic can be shown to be

$$\left(p + \frac{a}{v^2}\right)(v - b)^k = \text{const.}$$

where  $k$  is the ratio of the specific heats at constant pressure and constant volume. The slope of this curve is then found to be

$$\frac{dp}{dv} = \frac{2a}{v^3} - \frac{k(p + a/v^2)}{v - b},$$

which becomes, substituting the values of the critical pressure and volume

$$\left(\frac{dp}{dv}\right)_{\text{crit.}} = \frac{2a(1-k)}{27b^3},$$

which is negative for real positive values of  $a$  and  $b$ . Hence the isentropic appears also to cut through the broken line, but less sharply.

Still further, we believe that the line of constant volume does not again pass out of this non-homogeneous or unstable area, while the isentropic may. Hence it has seemed to me necessary

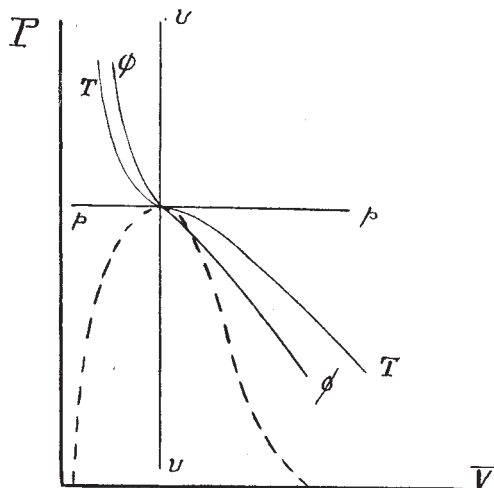


FIG. 1.

to consider the vertical line of Fig. 2 a line of constant volume, and the horizontal line an isentropic, while the critical point lies a little to the left of the vertex of the curve, so that the isentropic slightly cuts through the broken line.

The choice of co-ordinates will then be: energy, vertically downward, in the three-dimensional model, volume, measured to the right, in Maxwell's diagram, and entropy vertically downward in the same diagram. This choice is not inconsistent with the arrows in the upper left hand corner of the diagram. The model, which has been constructed in accordance with these considerations, is shown in the accompanying figure (Fig. 3). It satisfies the general requirements as to slope and convexity. It represents the gaseous or vapour state, as having in general the greatest volume and a great range of pressure, &c.

One property, however, does not seem to be indicated by this model, nor do I see how to satisfactorily change it so that this can be done. It has been deduced mathematically and shown experimentally that if a saturated vapour be expanded adiabatically, or isentropically, it may either become superheated, or partially condense to liquid, in fact both phenomena can be shown with one substance, for instance, chloroform above 127° C. becomes superheated, and below this temperature no visible effect is produced by either expansion or compression. That is, there is an isentropic which is at a particular point tangent to the "steam line," those on one side of it not touching it at all,

while those on the other cut both in and out. This particular property is not shown by the model as constructed, with the present choice of co-ordinates. If, however, we had measured entropy horizontally in the diagram, then the isentropics, being vertical, might be tangent to cut through the steam-line. This choice of co-ordinates has, however, seemed impossible for the reasons previously given.

We may if we wish discuss the question by a different method. The lines drawn in Figs. 1 and 2 are all lines through the critical point. In Fig. 1 the lines of constant pressure and

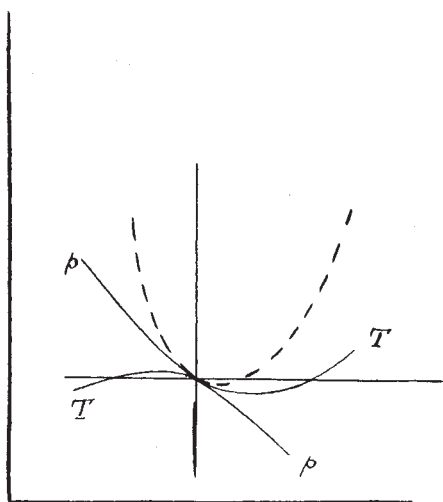


FIG. 2.

temperature are tangent to the broken line; Fig. 2 shows the same property. In Fig. 1 the line of constant entropy cuts the broken line twice, but no other pair of lines has more than one intersection. Fig. 2 does not, as drawn, show the same property. In Fig. 1, passing from the water-line around the critical point in the homogeneous region to the steam-line, one cuts the lines in the following order: water-line, pressure, temperature, entropy, volume, pressure, temperature, entropy, steam-line. Fig. 2 gives the same order, with the choice of co-ordinates, which we have adopted, if we let the temperature lines always

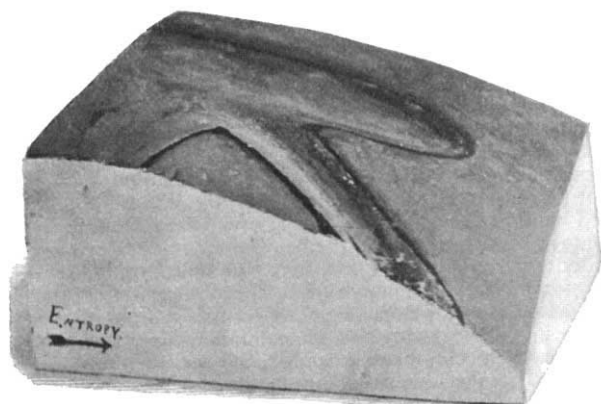


FIG. 3.

slope downward, as do the pressure lines. With this change the two diagrams seem to agree, but otherwise their disagreement seems hopeless.

I shall be very glad to receive from any one any suggestion which will help to remove the apparent disagreement between the two diagrams, or so modify the model that it may more completely represent the possible properties of actual bodies than it now seems to do.

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The University of California,  
Berkeley, Cal., U.S.A., February 1.

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### To Calculate a Simple Table of Logarithms.

A YEAR ago Prof. Perry drew attention to a method by which a schoolboy knowing how to extract square roots could, with the help of squared paper, construct a table of logarithms (NATURE, February 23, 1899).

It does not appear to be known that it is possible for a boy to make a simple table of logarithms in a few minutes without even knowing square root in arithmetic.

Up to a few years ago the teaching of logarithms in schools was generally deferred until they were required in trigonometry for the solution of triangles, but the general introduction of practical physics into secondary schools has resulted in the teaching of logarithms to younger boys.

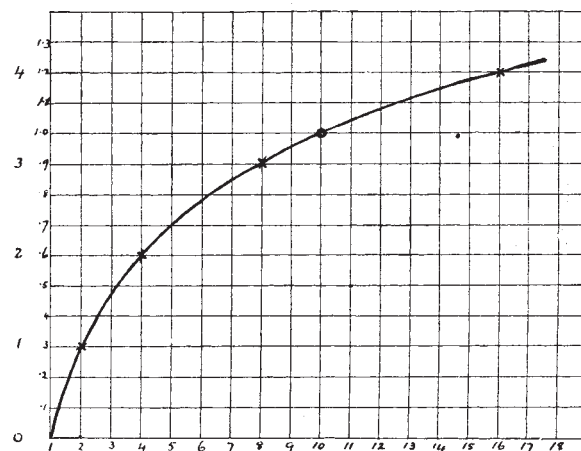
The following method which I have introduced into several Schools of Science in my district has been carefully tested in classes of boys of about thirteen years of age with excellent results.

On a sheet of squared paper ruled in inches and tenths, plot logarithms to base 2:  $\log 2 = 1$ :  $\log 4 = 2$ :  $\log 8 = 3$ :  $\log 16 = 4$ , and draw a curve.

It will be found convenient to arrange numbers from 1 to 16 on a horizontal axis, taking 1" as unit, and the logarithms on the vertical axis, taking 3" as unit.

From the curve read off the value of  $\log_2 10$ , which will be found to be approximately  $3\frac{1}{3}$ . Let us assume that  $\log_2 10$  is exactly  $3\frac{1}{3}$ .

On any system of logarithms  $\log 4 = 2$ :  $\log 2 = 1$ :  $\log 8 = 3$ :  $\log 16 = 4$ , &c. Hence the curve obtained may be used to represent



The left-hand vertical column of figures in the diagram represent scale logs. to base 2, and the right-hand column scale logs to base 10.

logarithms to any base if the ordinates are measured on a suitable scale.

The scale used for measuring logs. to base 2 is a plain scale. To construct a scale for measuring logs. to the base 10, write  $\log 10 = 1$  instead of  $3\frac{1}{3}$ ; and as this falls on the 10th line, the distance from 0 to 1 can be at once divided into 10 parts, and numbered 0.1: 0.2, &c., the finer lines (not shown in the diagram) giving the second decimal place.

Having assumed that  $\log_2 10 = 3\frac{1}{3}$ ,  $\log_{10} 2$  becomes .300 instead of .301, so that the values from the curve are in error to the extent of  $1/300$ ; but this is not greater than small errors due to the freehand drawing of the curve and irregularities in the ruling of the squared paper.

ARTHUR DUFTON.

Sheffield, February 13.

THE publication of Mr. Dufton's method will, I think, serve a useful purpose. It is a common exercise in schools to plot on squared paper, numbers and their logarithms to the base 2 (see Blaine's "Methods of Calculating," Spon), to give a general notion of how the logarithm varies as the number varies; but I have never known it to be made a method of calculation. Indeed, I do not think it right to give a boy the idea that he may find  $\log_{10} 10$  by interpolation between  $\log_2 8$  and  $\log_2 16$ . There is a specious appearance of accuracy due to the fact that  $\log_2 10$  is so nearly  $3\frac{1}{3}$ ; and Mr. Dufton heightens it by using